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THE METHOD OF LINES FOR THE ANALYSIS OF PLANAR WAVEGUIDES WITH MAGNETIZED FERRITE SUBSTRATE

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Abstract

It is shown, how the very efficient method of lines can be applied to the analysis of planar waveguides (e.g. microstrip, slotline, finline or more complex structures) on ferrite substrate transversely magnetized perpendicular to the plane of the substrate. Dispersion diagrams for microstrip on magnetized ferrite substrate obtained with a desktop computer are presented.

Introduction

The method of lines has been proved to be very efficient [1,2] for calculating the characteristic properties of planar microwave waveguides, e.g. microstrip, slotline and finline or other structures built from them e.g. resonators, periodic structures [3] . The method has no problems with the relative convergence phenomenon, is very accurate and has low memory capacity requirements, so that even relatively complex structures can be analyzed on a desktop computer.

In this contribution it will be shown, how the method of lines can be applied to shielded planar structures on ferrite substrate transversely magnetized perpendicular to the plane of the substrate. The much simpler case with transversely magnetized ferrite in the plane of the substrate is given in [4] . The structure here may contain an arbitrary number of dielectric and magnetic layers, but the principle is demonstrated for the two-layer structure in fig. 1 . Because the boundary conditions at electric or magnetic side walls in this case are not homogeneous in the ferrite layer ist is assumed that the structure in fig. 1 is an elementary cell of a periodic structure in x-direction.

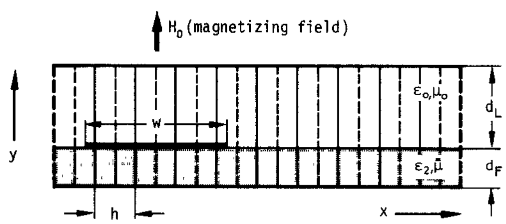

Fig. 1 Cross-section of planar microwave structure on magnetized ferrite substrate (—— lines for Ey ---- lines for Hy)

The computed dispersion diagrams are compared with results of Krause [5] , who used a mode matching procedure.

Method of analysis

In the ferrite substrate Maxwell's equations for time-harmonic electromagnetic fields can be written as

$$\nabla \times H = j\omega\varepsilon\, E \qquad \nabla \cdot (\bar{\bar{\mu}} H) = 0$$
$$\nabla \times E = -j\omega\bar{\bar{\mu}}\, H \qquad \nabla \cdot E = 0 \tag{1}$$

With the d.c. magnetic field H_0 in y-direction the permeability tensor becomes

$$\bar{\bar{\mu}} = \mu_0 \begin{bmatrix} \mu_1 & 0 & j\mu_2 \\ 0 & 1 & 0 \\ -j\mu_2 & 0 & \mu_1 \end{bmatrix}$$

with

$$\mu_1 = 1 + \frac{\gamma^2 H_0 M_s}{(\gamma H_0)^2 - \omega^2} \quad , \quad \mu_2 = \frac{\omega \gamma M_s}{(\gamma H_0)^2 - \omega^2}$$

γ is the gyromagnetic ratio and M_s the saturation magnetization.

The propagation is assumed to be in the z-direction with exp($-j\beta z$). According to [6] the electromagnetic field can be determined from the components in the direction of the d.c. magnetic field. The following equations are obtained for the field components E_y and H_y:

$$\frac{\partial^2 Ey}{\partial x^2} + \frac{\partial^2 Ey}{\partial y^2} - \beta^2\, Ey + \mu_f k_f^2\, Ey = k_f\, \eta\, \frac{\mu_2}{\mu_1} \frac{\partial Hy}{\partial y} \tag{3}$$

$$\frac{\partial^2 Hy}{\partial x^2} + \frac{1}{\mu_1} \frac{\partial^2 Hy}{\partial y^2} - \beta^2\, Hy + k_f^2\, Hy = -\frac{k_f}{\eta} \frac{\mu_2}{\mu_1} \frac{\partial Ey}{\partial y} \tag{4}$$

with

$$\mu_f = \mu_1 - \mu_2^2/\mu_1 \ , \ k_f^2 = \omega^2 \varepsilon_2 \mu_0 , \ \eta = (\mu_0/\varepsilon_2)^{1/2}$$

The boundary conditions on a wall in the y-z-plane are as follows:

Electrical wall:

$$Ey(=Ez) = 0; \quad \frac{\partial Hy}{\partial x} = -j\, \frac{\mu_2}{\mu_1} \frac{\partial Hz}{\partial y} \tag{5}$$

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Magnetic wall:

$$H_y(=H_z) = 0\ ;\quad \frac{\partial E_y}{\partial x} = -\mu_2\,\eta\,k_f\,H_x \tag{6}$$

In order to solve these equations, the fields are phase normalized by multiplying with e^{jbx} and discretized with respect to the x-variable with interval size h (fig.1). The discretization lines for Ey and Hy are shifted with respect to each other in order to obtain smaller discretization erros [1]. The partial derivatives with respect to x can be written as

$$h e^{jbx}\frac{\partial E_y}{\partial x} \to [D][Se]\vec{E}_y\ ;[Se] = \mathrm{diag}[e^{jb\,ih}]$$
$$h e^{jbx}\frac{\partial H_y}{\partial x} \to -[D]^{*t}[Sh]\vec{H}_y\ ;[Sh] = \mathrm{diag}[e^{jbh(i+\frac{1}{2})}] \tag{7}$$

$$h^2 e^{jbx}\frac{\partial^2 E_y}{\partial x^2} \to -[D]^{*t}[D][Se]\vec{E}_y$$
$$h^2 e^{jbx}\frac{\partial^2 H_y}{\partial x^2} \to -[D][D]^{*t}[Sh]\vec{H}_y \tag{8}$$

where $\vec{H}y$ and $\vec{E}y$ are the column vectors of dicretized fields. The lateral boundary conditions must be included in the difference-operators. Because of the partially nonhomogeneous conditions in eq.(5) and (6) this is not possible in simple way. Therefore periodic conditions are assumed, that means that according to Floquets theorem we introduce

$$F_{i+p} = F_i e^{-jbhp} = F_i S_1^{-p} \tag{9}$$

where F_i stands for anyone of the field components at line i and ph is the lenght of the period. bhp may be chosen to 0 or π. The difference-operators are given now as

$$[D] = \begin{bmatrix} -S & S^* & & \\ & \ddots & \ddots & \\ & & \ddots & S^* \\ S^* & & & -S \end{bmatrix} ;\ [D][D]^{*t} = \begin{bmatrix} 2 & -S_1^* & & -S_1 \\ -S_1 & \ddots & \ddots & \\ & \ddots & \ddots & -S_1^* \\ -S_1^* & & -S_1 & 2 \end{bmatrix} \tag{10}$$

with $\quad S_1 = S^{*2}$.

By means of the orthogonal transformation matrices [Te] and [Th], of which the columns are the eigenvectors of the matrices $[D]^{*t}[D]$ and $[D][D]^{*t}$ respectively [1], the vectors of the discretized and phase normalized fields are transformed into the spectral domain

$$[Se]\vec{E}_y = [Te]\underline{E}_y \text{ and } [Sh]\vec{H}_y = [Th]\underline{H}y \tag{11}$$

with $\quad [Te,h]^{*t}[D][D]^{*t}[Te,h] = \mathrm{diag}[\lambda^2]$

$[Te]_{i,k} = \exp(j2\pi ik/p)\sqrt{p}$

$[Th]_{i,k} = \exp(j2\pi(i+.5)k/p)/\sqrt{p}$

It is found, that

$[Te,h]^*[D][Te,h] = [\lambda]$ and

$[Th]^{*t}[D][Te] = j[\lambda]$

Transforming equations (3) and (4) yields a system of ordinary differential equations

$$\left(\frac{d^2}{dy^2} + \mu_f k_f^2 - \beta^2\right) - \frac{1}{h^2}[\lambda^2]\Big)\underline{E}_y = k_f\,\eta\,\frac{\mu_2}{\mu_1}\frac{\partial}{\partial y}\underline{H}y \tag{12}$$

$$\left(\frac{d^2}{\mu_1 dy^2} + k_f^2 - \beta^2\right) - \frac{1}{h^2}[\lambda^2]\Big)\underline{H}y = -\frac{k_f}{\eta}\frac{\mu_2}{\mu_1}\frac{\partial}{\partial y}\underline{E}y \tag{13}$$

that can be solved entirely in the spectral domain for each pair of elements Ey_i, Hy sperately. By combination a system of uncoupled fourth order differential equations for $\underline{E}y$ or $\underline{H}y$ can be derived:

$$\left(\left(\frac{1}{\mu_1}\frac{d^2}{dy^2} + [\xi^h]\right)\left(\frac{d^2}{dy^2} + [\xi^e]\right) + k_f^2\frac{\mu_2^2}{\mu_1^2}\frac{d^2}{dy^2}\right)(\underline{E}y \text{ or } \underline{H}y) = 0 \tag{14}$$

with

$$[\xi^h]_i = k_f^2 - \beta^2 - \frac{1}{h^2}[\lambda^2]_i\ ,\ [\xi^e]_i = \mu_f k_f^2 - \beta^2 - \frac{1}{h^2}[\lambda^2]_i$$

The solution of eqn.(14) is obtained as a superposition of two partial sulutions.

$$\underline{E}y_i = A_{1i}\cosh k_{y1i}\,y + A_{2i}\cosh k_{y2i}\,y \tag{15}$$

$$\underline{H}y_i = B_{1i}\sinh k_{y1i}\,y + B_{2i}\sinh k_{y2i}\,y \tag{16}$$

with

$$2\cdot(k_{y1,2}^2)_i = -\mu_1\left(x \pm\sqrt{x^2 - 4\frac{\xi^e\xi^h}{\mu_1}}\right)_i$$

$$x_i = \left(\xi^h + \frac{1}{\mu_1}\xi^e + k_f^2\frac{\mu_2^2}{\mu_1^2}\right)_i$$

Because of eqn.(12) and (13) the coefficients B_{1i} and B_{2i} are determined by A_{1i} and A_{2i}.

All other components in the ferrite substrate can now be expressed in terms of Ey_i and Hy_i and their first derivatives with respect to y.

In the dielectric layers the electromagnetic field is obtained in a similiar but simpler manner with $\mu_2 = 0$ and $\mu_1 = 1$.

Applying continuity conditions at the ferrite-dielectric interface yields a system of equations for the tangential electric field components and the current density distributions at the interface in the spectral domain

$$[Z]\begin{bmatrix} \underline{j}z \\ \underline{j}x \end{bmatrix}_{y=d_F} = \begin{bmatrix} \underline{E}z \\ \underline{E}x \end{bmatrix}_{y=d_F}$$

where $[\underline{Z}]$ contains for our simple structure four diagonal submatrices.
The final boundary condition, $E_{tan} = 0$ on the metallic parts of the interfaces or $J_{tan} = 0$ in the slots in a more complex structure must be imposed in the original domain, so that an inverse transformation is required. Only a few lines are required trough the strips or slots. Thus, the final eigenvalue equation has a very low order and the propagation constant can be determined with relatively small effort.

Numerical results

For the microstrip shown in fig.(1) some numerical results are obtained. The discretization distance h is chosen such, that the strip width equals $W = (M - .5)h$, where M is the number of Ey-lines on the strip. The dispersion diagram for the fundamental mode fig. (2) was calculated with M = 4. The agreement with the results of Krause [5] (dashed curves) is good in the case of the normalized magnetizing field $h_0^i = 8.1$ but for $h_0^i = 2.0$ the difference increases with frequency.

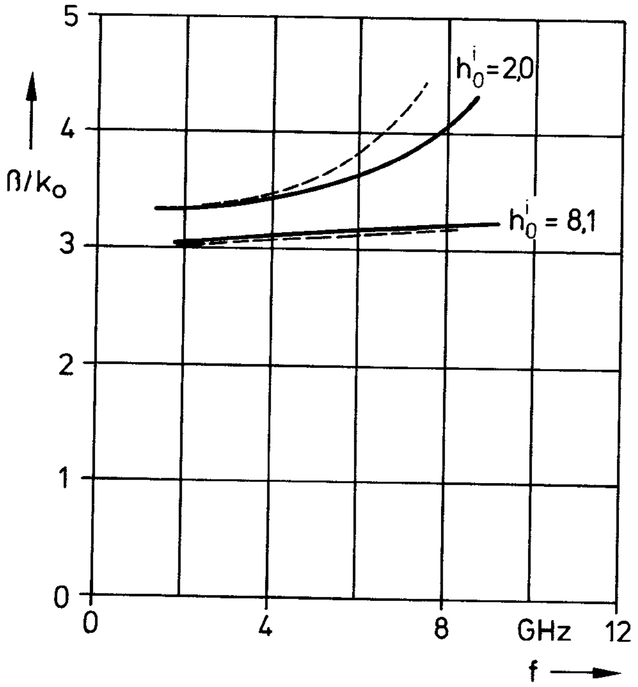

Fig. 2 Dispersion diagram for a microstrip on magnetized ferrite (——— this method
----- from [5])
$W = 1mm$, $d_F = .635$ mm, $d_L = 5\ d_F$
$\varepsilon_{r2} = 12.3$, $M_s = 1.4$ kA/cm, $g = 1.98$
$h_0^i = H_0/M_s$

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